Quantitative Finance

Economics, Finance and Management

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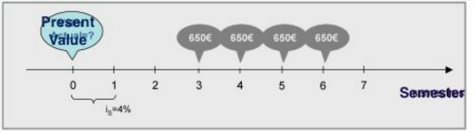
Example (Deferred Annuity)



$$PV = 650\ {}_{2|}a_{\overline{4}|i} = 650\ {}_{3|}\ddot{a}_{\overline{4}|i}$$

Definition (Deferred Annuity)

It's an annuity under which the first payment occurs at some specified future time.
The *PV* (Present Value) of an **annuity-due deferred** k **years**, with term R, is given by

$$\begin{aligned} PV &= R\ {}_{k|}\ddot{a}_{\overline{n}|i} = R\, v^k\, \ddot{a}_{\overline{n}|i} \\ &= R\ {}_{k-1|}a_{\overline{n}|i} = R\, v^{k-1}\, a_{\overline{n}|i} \end{aligned}$$

Example (Ordinary *vs* deferred)

Ordinary annuity

0	1	2	3	4	5	6
	500	500	500	500	500	500

$$PV_0 = 500\, a_{\overline{6}|1\%} = 2869.05; \quad FV_6 = 500\, s_{\overline{6}|1\%} = 3076.01$$

Deferred annuity

0	1	2	3	4	5	6	7	8
			500	500	500	500	500	500

$$PV_0 = 500\, a_{\overline{6}|1\%} \times 1.01^{-2} = 2840.64$$

$$FV_8 = 500\, s_{\overline{6}|1\%} = 3076.01$$

Definition (Perpetuity)

A Perpetuity is an anunuity with infinite term

Ordinary Perpetuity

$$a_{\overline{\infty}|i} = \lim_{n\uparrow\infty} a_{\overline{n}|i} = \lim_{n\to\infty} \frac{1-(1+i)^{-n}}{i} = \lim_{n\to\infty} \frac{1}{i}\left(1 - \frac{1}{(1+i)^n}\right) = \frac{1}{i}$$

Perpetuity-due

$$\ddot{a}_{\overline{\infty}|i} = 1 + \frac{1}{i}$$

Example

Perpetua will start studying at ULisboa, where she intends to stay and find a job after. $i_A = 0.05$. Options: a) Flat rent: €850/month; b) Buy flat: €127,500.

$$\begin{aligned} i_M &= (1,05)^{1/12} - 1 = 0,00407 \to 0,4074\% \\ PV = & \quad 850 a_{\overline{\infty}|i_M} = 208,640,16€ > 127,500€ \end{aligned}$$